EE 215 Fundamentals of Electrical Engineering — Spring 2010

Problem #1 Solution

P 1.10 First we use Eq. (1.2) to relate current and charge:

$$i = \frac{dq}{dt} = 20\cos 5000t$$

Therefore, $dq = 20\cos 5000t\,dt$

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 20\int_0^t \cos 5000y\,dy$$

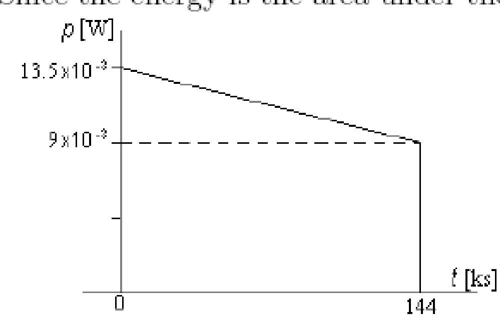
We solve the integral and make the substitutions for the limits of the integral, remembering that $\sin 0 = 0$:

$$q(t) - q(0) = 20\frac{\sin 5000y}{5000}\Big|_0^t = \frac{20}{5000}\sin 5000t - \frac{20}{5000}\sin 5000(0) = \frac{20}{5000}\sin 5000t$$

But $q(0) = 0$ by hypothesis, i.e., the current passes through its maximum value at $t = 0$, so $q(t) = 4\times 10^{-3}\sin 5000t\ \text{C} = 4\sin 5000t\ \text{mC}$

P 1.13 $p = vi;\qquad w = \int_0^t p\,dx$

Since the energy is the area under the power vs. time plot, let us plot p vs. t.



Note that in constructing the plot above, we used the fact that 40 hr = 144,000 s = 144 ks

$$p(0) = (1.5)(9\times 10^{-3}) = 13.5\times 10^{-3}\ \text{W}$$

$$p(144\ \text{ks}) = (1)(9\times 10^{-3}) = 9\times 10^{-3}\ \text{W}$$

$$w = (9\times 10^{-3})(144\times 10^3) + \frac{1}{2}(13.5\times 10^{-3} - 9\times 10^{-3})(144\times 10^3) = 1620\ \text{J}$$

P 1.14 Assume we are standing at box A looking toward box B. Then, using the passive sign convention $p = -vi$, since the current i is flowing into the $-$ terminal of the voltage v. Now we just substitute the values for v and i into the equation for power. Remember that if the power is positive, B is absorbing power, so the power must be flowing from A to B. If the power is negative, B is generating power so the power must be flowing from B to A.

[a] $p = -(125)(10) = -1250$ W 1250 W from B to A

[b] $p = -(-240)(5) = 1200$ W 1200 W from A to B

[c] $p = -(480)(-12) = 5760$ W 5760 W from A to B

[d] $p = -(-660)(-25) = -16{,}500$ W 16,500 W from B to A

P 1.17 [a] $p = vi = (0.05e^{-1000t})(75 - 75e^{-1000t}) = (3.75e^{-1000t} - 3.75e^{-2000t})$ W

$$\frac{dp}{dt} = -3750e^{-1000t} + 7500e^{-2000t} = 0 \qquad \text{so} \qquad 2e^{-2000t} = e^{-1000t}$$

$$2 = e^{1000t} \qquad \text{so} \qquad \ln 2 = 1000t \qquad \text{thus} \qquad p \text{ is maximum at } t = 693.15\,\mu\text{s}$$

$$p_{\max} = p(693.15\,\mu\text{s}) = 937.5 \text{ mW}$$

[b] $$w = \int_0^\infty [3.75e^{-1000t} - 3.75e^{-2000t}]\,dt = \left[\frac{3.75}{-1000}e^{-1000t} - \frac{3.75}{-2000}e^{-2000t}\Big|_0^\infty\right]$$

$$= \frac{3.75}{1000} - \frac{3.75}{2000} = 1.875 \text{ mJ}$$

P 1.24 [a]
$$\begin{aligned} q &= \text{area under } i \text{ vs. } t \text{ plot} \\ &= \left[\tfrac{1}{2}(5)(4) + (10)(4) + \tfrac{1}{2}(8)(4) + (8)(6) + \tfrac{1}{2}(3)(6)\right] \times 10^3 \\ &= [10 + 40 + 16 + 48 + 9]10^3 = 123{,}000 \text{ C} \end{aligned}$$

[b]
$$\begin{aligned} w &= \int p\,dt = \int vi\,dt \\ v &= 0.2 \times 10^{-3}t + 9 \qquad 0 \le t \le 15 \text{ ks} \end{aligned}$$

$0 \le t \le 4000s$

$$\begin{aligned} i &= 15 - 1.25 \times 10^{-3}t \\ p &= 135 - 8.25 \times 10^{-3}t - 0.25 \times 10^{-6}t^2 \\ w_1 &= \int_0^{4000} (135 - 8.25 \times 10^{-3}t - 0.25 \times 10^{-6}t^2)\,dt \\ &= (540 - 66 - 5.3333)10^3 = 468.667 \text{ kJ} \end{aligned}$$

$4000 \le t \le 12{,}000$

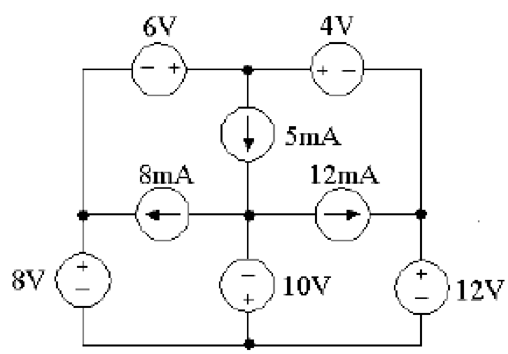
$$\begin{aligned} i &= 12 - 0.5 \times 10^{-3} t \\ p &= 108 - 2.1 \times 10^{-3} t - 0.1 \times 10^{-6} t^2 \\ w_2 &= \int_{4000}^{12{,}000} (108 - 2.1 \times 10^{-3} t - 0.1 \times 10^{-6} t^2)\, dt \\ &= (864 - 134.4 - 55.467)10^3 = 674.133 \text{ kJ} \end{aligned}$$

$12{,}000 \le t \le 15{,}000$

$$\begin{aligned} i &= 30 - 2 \times 10^{-3} t \\ p &= 270 - 12 \times 10^{-3} t - 0.4 \times 10^{-6} t^2 \\ w_3 &= \int_{12{,}000}^{15{,}000} (270 - 12 \times 10^{-3} t - 0.4 \times 10^{-6} t^2)\, dt \\ &= (810 - 486 - 219.6)10^3 = 104.4 \text{ kJ} \\ w_T &= w_1 + w_2 + w_3 = 468.667 + 674.133 + 104.4 = 1247.2 \text{ kJ} \end{aligned}$$

P 2.2 The interconnection is not valid. Note that the 10 V and 20 V sources are both connected between the same two nodes in the circuit. If the interconnection was valid, these two voltage sources would supply the same voltage drop between these two nodes, which they do not.

P 2.5



The interconnection is invalid. The voltage drop between the top terminal and the bottom terminal on the left hand side is due to the 6 V and 8 V sources, giving a total voltage drop between these terminals of 14 V. But the voltage drop between the top terminal and the bottom terminal on the right hand side is due to the 4 V and 12 V sources, giving a total voltage drop between these two terminals of 16 V. The voltage drop between any two terminals in a valid circuit must be the same, so the interconnection is invalid.

P 2.12 The resistor value is the ratio of the power to the square of the current: $R = \frac{p}{i^2}$. Using the values for power and current in Fig. P2.12(b),

$$\frac{5.5 \times 10^{-3}}{(50 \times 10^{-6})^2} = \frac{22 \times 10^{-3}}{(100 \times 10^{-6})^2} = \frac{49.5 \times 10^{-3}}{(150 \times 10^{-6})^2} = \frac{88 \times 10^{-3}}{(200 \times 10^{-6})^2}$$

$$= \frac{137.5 \times 10^{-3}}{(250 \times 10^{-6})^2} = \frac{198 \times 10^{-3}}{(300 \times 10^{-6})^2} = 2.2\ \text{M}\Omega$$

Note that this is a value from Appendix H.